Acoustic Firearm Discharge Detection and Classification in an Enclosed Environment

Lorenzo Luzi[1], Eric Gonzales[1], Paul Bruillard[1], James Skorpik[1], Michael Hughes[1]
[1]*Pacific Northwest National Laboratory*

John E. M^{c}Carthy[2],
[2]*Department of Mathematics, Washington University in Saint Louis*
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Two different signal processing algorithms are described for detection and classification of acoustic signals generated by firearm discharges in small enclosed spaces. The first is based on the logarithm of the signal energy. The second is a joint entropy. The current study indicates that a system using both signal energy and joint entropy would be able to both detect weapon discharges and classify weapon type, in small spaces, with high statistical certainty.

I. INTRODUCTION

We report on methods to both detect and classify firearm discharges in small, enclosed, environments with high statistical certainty. Some algorithms reported on here are capable of running on an embedded microcontroller system (Texas Instruments FRAM microcontroller unit number MSP430FR5989) that, with an associated microphone (InvenSense INMP404ACEZ-R7 microphone), is capable of signal acquisition and analysis. Moreover, such a system and software are suitable for wide-scale deployment in classrooms, movie theaters, and other public gathering places. Results for a proof-of-principle prototype are shown in Fig. (1). The purpose of this paper is to explain the acquisition and analysis of data such as that shown in the figure. In addition, we have made improvements on the algorithms in the embedded controller and investigated new algorithms which increase the statistical separation between different types of signals.

Implementation on a microcontroller limits the range of algorithms that may be employed. In addition, we have found that the governing dynamics of acoustic propagation are highly nonlinear. Consequently, we have focused on approaches that reduce a waveform, or a subsegment of a waveform $f(t)$, to a single number. This number, or receiver value, is then intended to be used as the basis for signal identification. Signal energy, or its logarithm, denoted $\log[E_f]$, combined with careful signal filtering has been shown to provide a good balance between computational complexity and statistical sensitivity. Our results show that signal energy analysis is able to clearly discriminate between firearm discharges and other acoustic background events, but not necessarily between firearm types.

To address the need to distinguish between firearm types, we have investigated various entropies as previous studies have demonstrated their utility for analysis of ultrasonic signals in both materials characterization and medical ultrasonics.[1–13] The current study demonstrates the utility of entropies for analysis of acoustic signals in the audio range as well. When used in conjunction with energy analysis it appears that acoustic discrimination of weapon type is also possible.

There is an extensive literature on firearm discharge detection and source identification. Previous investigations of firearm discharge have focused on outdoor source location,[14–16] and identification of firearm and ammunition type using shockwave analysis.[17,18] Other investigators have studied correlation and linear predictive coding for firearm detection and source recognition.[19] Threshold detection schemes using six different waveform characteristics, *e.g.*, magnitude of signal absolute value, median filter, Teager energy operator, correlation against a template, among others mainly for outdoor gunshot detection and source classification have been described by Chacn-Rodrguez *et al.*.[20] More generally, extraction of acoustic cues for forensic purposes has been investigated

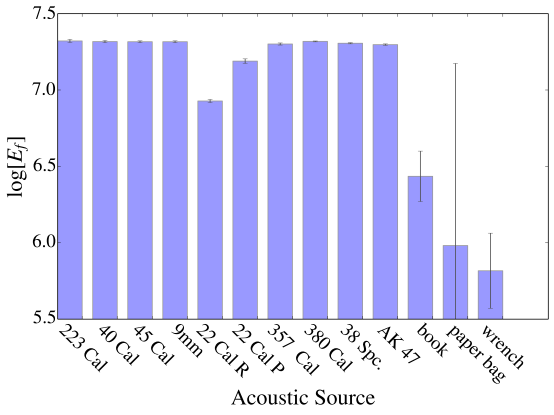


FIG. 1. The logarithm of the signal energy output from a prototype embedded sensor/processor for different acoustic sources. Error bars, indicate the standard deviations of N=five measurements for each acoustic source: for 223 caliber assault rifle (M16), 40 caliber pistol, 45 caliber pistol, 9 mm pistol, 22 caliber rifle, 22 caliber pistol, 357 caliber pistol, 380 caliber pistol, and AK47 assault rifle; N=4 for 38 special pistol, wrench tap; N=3 for book drop and N=2 air-filled paper bag "popping". The smaller N for the non-threat sources is the result of the sensor not triggering. Error bars are present on all bars but are very small in some cases. Not shown is a data from a test for false positives induced by the test-site fire alarm since this did not trigger the prototype.

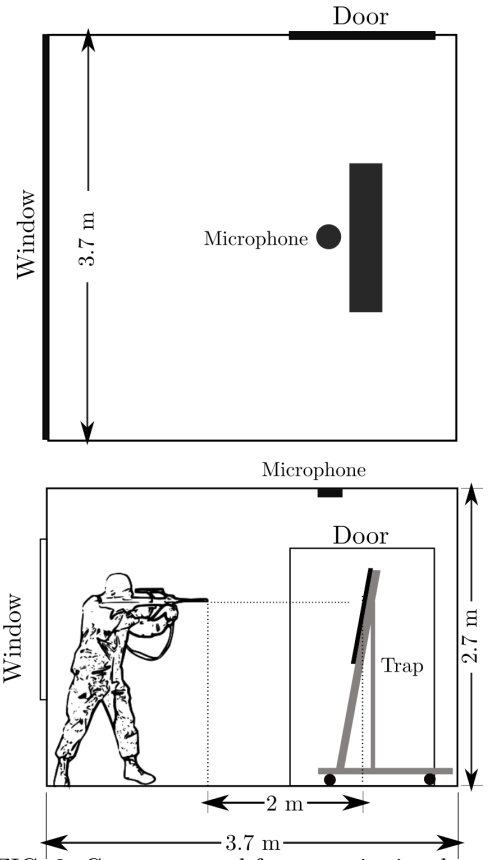


FIG. 2. Geometry used for acoustic signal acquisition.

by Hong and colleagues.[21]

The current study appears to be different from prior work in that it considers signals acquired indoors, in relatively small enclosures. Moreover, shockwave analysis or linear analysis techniques for source identification are not used at all.

II. DATA ACQUISITION

Two different groups of acoustic data were collected for this study. This first group consisted of "threat-type" signals, which were acquired by discharging several different firearms into a ballistic trap: 223 caliber automatic rifle (M-16 assault rifle; Fiocchi 45 grain frangible), 40 caliber semi-automatic pistol (Compact Smith & Wesson; S&W 125 grain frangible), 45 caliber automatic pistol (Taurus PT 145 PRO; Fiocchi 155 grain frangible), 9 mm semi-automatic pistol (Springfield Armory XD9; Fiocchi 100 grain luger) .22 rifle (Marlin Model 60; Remington 22LR 40 grain), 22 pistol (Intratec TEC-22; Remington 22LR 40 grain), 357 magnum (S&W Magnum; PMC 158 grain), 380 caliber pistol (Taurus PT 738; Fiocchi 380 auto), 38 special (S&W .38 Special AirLite; American Eagle 158 grain), and AK 47 (SAIGA 7.62x39 Legion; Tula 7.62x39R 124 HP). All firearms but the M16 where provided by the Kennewick, WA police department from their evidence locker, the M16 was provided by their special weapons and tactics team. This was done in order to sample an ensemble of acoustic sources that captures the variability found in non military/police arms. A second group of control data were also acquired. These consisted of "false" alarms: a book slapped on a table, an air-filled paper bag "popping", and a wrench striking a metal ladder.

Signals were collected in three different rooms of different dimensions: a large auditorium (12.8×8.4 m; height 2.7 m), a medium sized meeting room (4.6×6.1 m; height 2.7 m) and a small office (3.7×3.7 m; height 2.7 m). Acquisition of all data was completed during one 10 hour interval. Firearm discharge data were obtained with the assistance of the Kennewick, WA special weapons and tactics team, whose members volunteered to operate all firearms used in this study. We will report on results collected in the small room shown in Fig. (2) since this constitutes the most challenging environment in which to distinguish between threat and non-threat acoustic events and to discriminate between different types of threats. The figure shows the dimensions of the room, the placement of the microphones, location of trap and firearms operator. During acquisition of data, the door to the room was closed.

Acoustic signals were converted to electrical signals using a InvenSense INMP404ACEZ-R7 microphone connected to custom built amplifier circuitry. These signals were digitized, single-shot fashion, by a Teledyne LeCroy MSO 104MX$_s$B digital sampling oscilloscope to obtain raw data consisting of one million point waveforms (12-bit numbers; 1.0 MHz sampling rate). At least five waveforms for each type of acoustic source were acquired and stored for later analysis.

III. ANALYSIS

A. Experimental Nonlinearities

Several observations indicate that the experiment is governed, at least partially, by nonlinear dynamics. During acquisition of firearms signals, the marksman reported the ceiling tile immediately above the trap was being visibly displaced by the weapon discharge. Subsequent inspection of the digital waveforms on a fine time scale (not shown) reveals evidence of shock-like features.

As an additional check we acquired waveforms for a single and five-round discharge from the 223 caliber assault rifle. Figure (3) shows, in its top panel, the waveform recorded from the discharge of one round, $f_1(t)$. The waveform, $f_5(t)$, for five, automatic, discharges is shown in the middle panel. Both waveforms were Fourier transformed to obtain respectively, $F_1(\omega)$ and $F_5(\omega)$. If the propagation was linear, or even weakly linear, then the inverse Fourier transform of $F_5(\omega)/F_1(\omega)$ should produce a time series having five delta function spikes. To avoid

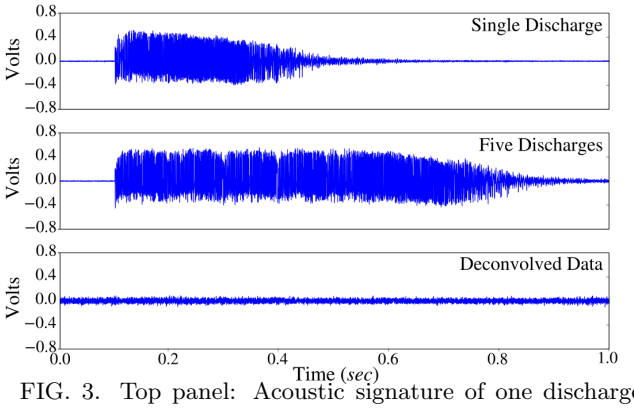


FIG. 3. Top panel: Acoustic signature of one discharge of 223 caliber assault rifle. Middle panel: acoustic signature of automatic discharge of five rounds. Bottom panel: The deconvolution, as discussed in connection with Eqs. (1) and (2) below, of the five discharge by the single discharge signature. In a linear system such a deconvolution should produce a series of five delta functions. The fact that these are not evident supports the conclusion that acoustic wave propagation is not linear in the current study.

division by zero we actually divide by a modified version of $F_1(\omega)$, specifically

$$\frac{F_5(\omega)}{\tilde{F}_1(\omega)}, \tag{1}$$

where

$$\tilde{F}_1(\omega) = \begin{cases} F_1(\omega) & \text{if } |F_1(\omega)| \geq \rho, \\ \rho & \text{if } |F_1(\omega)| < \rho, \end{cases} \tag{2}$$

and $\rho = 10^{-6}$ is a regularizing term. The inverse Fourier transform of this function was computed and is shown in the bottom panel of the Fig. (3). It is evident that there is no pulse train.

The strong non-linearities exhibited by the acoustics preclude identification of signal source using signal processing techniques based on linear systems theory such as matched filters. Instead, we will focus on techniques that take subsegments of the acoustic waveform and produce a single number or receiver value.

B. Energy and Entropy Analysis of Waveforms

To describe our analysis we will denote the acoustic waveforms captured by, $f(t)$. We will also employ the convention that the domain of $f(t)$ is $[0, 1]$.

For $f(t)$, the signal energy is

$$E_f = \int_0^1 f(t)^2 dt. \tag{3}$$

We may also compute a joint entropy of acoustic waveforms, $f(t)$, using a reference function $g(t)$. In the case where $f(t)$ and $g(t)$ are differentiable functions this entropy is given by[6]

$$H_{f,g} \equiv -\frac{1}{2}\int_0^1 dt\ \frac{\min[|f'(t)|, |g'(t)|]}{\max[|f'(t)|, |g'(t)|]} - \int_0^1 dt\ \log\left[\max[|f'(t)|, |g'(t)|]\right]. \tag{4}$$

The strategy for choosing the reference waveform, $g(t)$, in the case where subtle changes in $f(t)$ must be detected has been described elsewhere.[7] Although these techniques are not technically applicable for the present investigation, we will take them as an operational starting point. The justification for this approach is twofold. In previous studies,[8,9] it has been observed that the joint entropy calculation has many of the attributes of a matched filter. In particular, $H_{f,g}$ is often extremized for waveforms "close" to $f(t)$, when $g'(t)$ is a step-like function, with transitions located at the critical values of $f(t)$. In addition, Theorem 1 of a previous study of the variational properties of joint entropy[7] also suggests that this strategy would be successful. Consequently, we will use this approach, described in greater detail below (Fig. (6)) to discriminate between different classes of firearms.

C. Signal Preprocessing

The goal of this study was to discover a suite of signal receivers that are suitable for two different tasks: discharge detection *vs.* firearm identification. Consequently, after an initial "gating" operation to remove the noise-only pre-trigger portion of the digital waveform, two different preprocessing schemes were applied to the raw data, prior to computation of either signal energy or entropy.

For signal energy computations the data were decimated (*i.e.*, only every tenth point was kept) and then bandpass filtered to exclude frequencies outside of the range extending from 1 kHz to 26 kHz. This was accomplished in the frequency domain by multiplying the Fourier transform of the raw data by the conjugate symmetric form of

$$\frac{\left[\tanh\left(\frac{f-f_{l.b.}}{a}\right)+1\right]\left[\tanh\left(\frac{f_{u.b.}-f)}{a}\right)+1\right]}{4}, \tag{5}$$

where, $f_{l.b.} = 10^3$, $f_{l.b.} = 26 \times 10^3$, and the sharpness parameter for the filter was set to $a = 10$. All computations are performed using units of Hertz. The first 256 points of the acoustic waveform were then used to compute the logarithm of the signal energy according to Eq. (3). The analysis thus uses the first 2.56ms of the acoustic waveforms.

For entropy calculations a slightly different scheme was

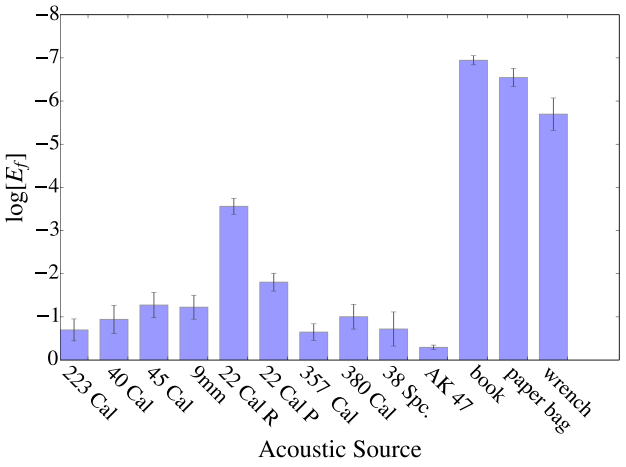


FIG. 4. Separation of "threat-type" versus "nonthreat-type" by the logarithm of signal energy, log[E_f].

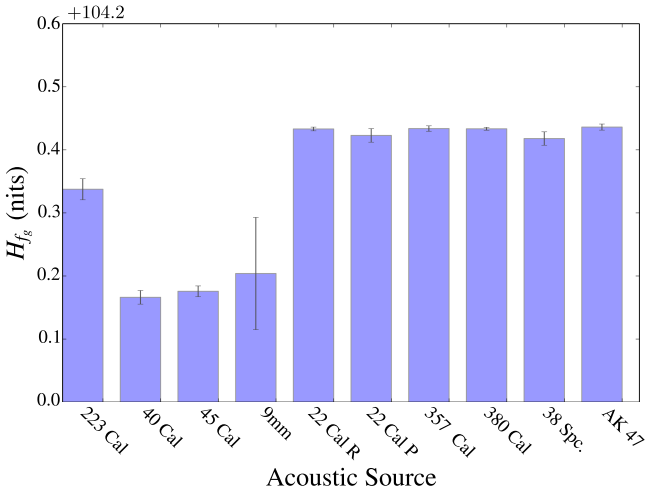


FIG. 5. Separation of the acoustic signatures of different firearms by joint entropy, $H_{f,g}$. The reference was computed using a step-like function with transitions at the extrema of one of the 223 assault rifle waveforms. Note vertical axis constant offset of 104.2.

TABLE I. Confidence ratios of "Threat-type" vs. "Nonthreat-type" sources using log[E_f]

	book	paperbag	wrench
223 caliber rifle	22.60	17.68	11.03
40 caliber pistol	17.50	14.43	9.57
45 caliber pistol	18.10	14.5	9.28
9 mm pistol	19.22	15.26	9.59
22 caliber rifle	15.81	10.63	5.11
22 caliber pistol	22.07	16.05	9.09
357 caliber pistol	28.66	20.69	12.00
380 caliber pistol	19.41	15.57	9.95
38 Special pistol	15.19	13.00	9.13
AK 47 rifle	55.76	28.77	14.28

used since it was found that better signal discrimination could be obtained using the initial portion of the signals. In this case the data were neither bandpass filtered nor decimated. In addition, the analysis uses only the first 32μs of the acoustic waveforms.

For comparison, the analysis parameters written to the embedded controller used to acquire the data shown in Fig. (1) are: bandpass filtering from 5 KHz to 26 KHz, eight-bit digitization of 98304 points with 7 μs per point.

IV. RESULTS

A. "Threat" *vs.* "Nonthreat"

Each of the waveforms acquired for each source type were analyzed as described above to obtain either log energy, log[E_f], or joint entropy, $H_{f,g}$. The mean and standard deviations of the ensemble for each source were then computed.

Figure (4) shows the results obtained for the log[E_f] analysis. The error bars in the plot are equal to one standard deviation. There is wide separation between the "threat-type" and "nonthreat-type" bars. However, there appears to be little separation between either the 223 caliber (M16) and AK47 ("long rifles") and any pistols. This would be useful information in certain circumstances.

In order to quantify this separation, the pair-wise differences between each acoustic source were computed along with associated standard deviation using the standard methods for error propagation[22]. From these, the mean difference, which can be either positive or negative, divided by the associated standard deviation was computed in order to obtain a noise-normalized measure of change between receiver values for different acoustic sources. This ratio, which we will use to quantify the sensitivity of analysis techniques, is often defined as the statistical confidence,[23] and is the reciprocal of the coefficient of variation of a random variable. Larger values are better as they imply greater statistical separation between random variables, in our case signal receivers. Small values suggest that the sources are statistically indistinguishable to the signal receiver.

The confidence values characterizing the relation between "threat-type" and "nonthreat-type"-signals are summarized in Table I. We observe that all confidences are larger than one, suggesting that these types of signals should be easily distinguished using only log[E_f]. Moreover, energy calculation is simple and well suited to our goal of reduction-to-practice on low cost hardware.

TABLE II. Confidence ratios for different "threat-type" sources obtained using $\log[E_f]$ analysis.

	223	40Cal	45Cal	9mm	22Cal(R)	22Cal(P)	357Cal	380Cal	38 Spc.	AK 47
223 rifle (M16)	-	-	-	-	-	-	-	-	-	-
40 cal. pistol	0.59	-	-	-	-	-	-	-	-	-
45 cal. pistol	1.48	0.76	-	-	-	-	-	-	-	-
9 mm pistol	1.40	0.66	0.12	-	-	-	-	-	-	-
22 cal. rifle	9.09	6.98	6.57	7.00	-	-	-	-	-	-
22 cal. pistol	3.37	2.23	1.47	1.67	6.32	-	-	-	-	-
357 cal.pistol	0.16	0.78	1.78	1.71	10.92	4.10	-	-	-	-
380 cal. pistol	0.80	0.14	0.66	0.55	7.49	2.26	1.03	-	-	-
38 Special pistol	0.04	0.44	1.13	1.05	6.50	2.43	0.16	0.58	-	-
AK 47 rifle	1.55	1.96	3.28	3.29	16.96	7.07	1.77	2.43	1.06	-

B. "Threat-Type" Discrimination

As mentioned previously, discrimination between "threat-type" waveforms would be useful information. Table II summarizes the absolute values of all confidence ratios obtained in pair-wise comparison of $\log[E_f]$ values for "threat-type" sources. Only the values below the diagonal are shown since the table is symmetric about this line. We observe that many entries are greater than one, suggesting that in many cases highly reliable discrimination between sources is possible. However, there are also many entries that are less than one. Particularly troubling is the fact that several of these entries appear in the first column indicating that $\log[E_f]$ provides poor discrimination between several pistols and the assault rifle.

Consequently, we have investigated the use of different joint entropies, H_{f,g_i}, as an additional tool for weapons identification. Each reference function, $g_i(t)$, was generated using one of the "threat-type" waveforms according to the methods described previously[7], for instance $g_1(t)$ was computed using one of the 223 rile waveforms, $g_2(t)$ was computed using one of the 40 caliber waveforms and so forth with $g_{10}(t)$ being computed using one of the AK 47 rifle waveforms. For completeness, we illustrate this computation in Fig. (6). The line with long and short dashes shows the a portion of the bandpass filtered version of the underlying waveform coincident with the onset of the acoustic pulse generated by discharge of the 223 assault rifle. Zero time indicates the point at which the LeCroy MSO 104MX$_s$B digital sampling oscilloscope triggered during data acquisition when the incoming voltage crossed the threshold level of roughly 125 mV. The 32μs segment of this waveform, which has been selected for H_{f,g_i} analysis is shown by the solid line. Solid black circles indicate the locations of the start of this segment and its extrema. The dashed step-like function shows a scaled version of the resulting $g_1'(t)$, which had high values of 10000 and low values of 0.001.

An example plot of the entropies H_{f,g_1}, along with associated standard deviation bars, obtained using the reference function generated using a 223 caliber waveform is shown in Fig. (5). The figure shows a clear separation between the "long rifle" 223 data and the "pistol" 40 caliber, 45 caliber and, to a lesser extent, 9 mm data. For this plot, the confidence ratios quantifying the separation between the 223 (a "long rifle") and the 357 caliber and 380 caliber pistols data improves from the Table II values of 0.16 and 0.80 to 5.58 and 5.69 respectively. However, for the 22 caliber rifle to confidence is decreased from its Table II value of 9.09 to 5.64.

To be thorough, we have calculated confidence tables for joint entropy analogous to the Table (II) using a representative of each type of "threat" waveform, *i.e.*, for all $g_i(t), i = 1, \ldots, 10$. To assess the sensitivities obtainable using this suite of signal receivers, the maximum absolute values for each entry in these over all ten tables have been collected and are shown in Table III. We observe that the where the entries of this table are low, the corresponding entries of Table II are high and *visa versa*.

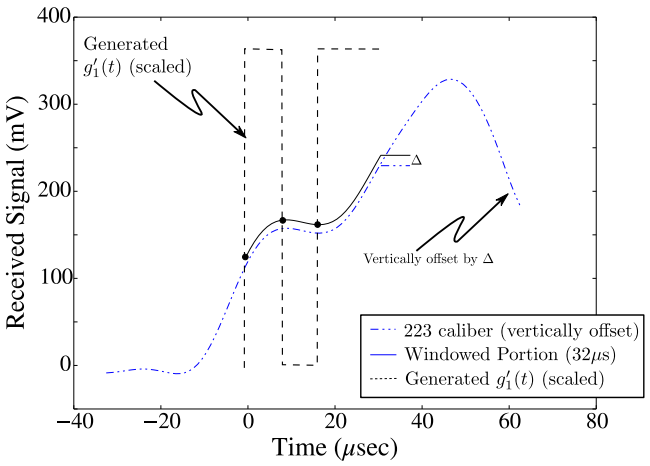


FIG. 6. An illustration of the steps used to calculate a reference waveform for the 223 caliber assault rifle. The dashed step-like function is the derivative of the calculated reference function $g_1'(t)$, which is shown instead of the reference $g_1(t)$, since its relation to the extrema of 32μs segment is more easily visualized.

TABLE III. Maximum confidence ratios for different "threat-type" sources obtained using $H_{f,g}$ analysis for $g_i(t)$ derived from different acoustic sources as described in section IV B.

	223	40Cal	45Cal	9mm	22Cal(R)	22Cal(P)	357Cal	380Cal	38 Spc.	AK 47
223 rifle (M16)	-	-	-	-	-	-	-	-	-	-
40 cal. pistol	8.61	-	-	-	-	-	-	-	-	-
45 cal. pistol	8.63	1.42	-	-	-	-	-	-	-	-
9 mm pistol	1.48	1.00	0.65	-	-	-	-	-	-	-
22 cal. rifle	5.64	53.46	29.92	7.40	-	-	-	-	-	-
22 cal. pistol	4.30	38.33	18.00	9.33	1.16	-	-	-	-	-
357 cal.pistol	5.58	40.44	27.67	9.71	0.91	1.74	-	-	-	-
380 cal. pistol	5.69	59.04	31.65	9.80	1.24	1.94	0.58	-	-	-
38 Special pistol	4.06	16.59	17.75	6.01	1.37	0.48	1.37	1.42	-	-
AK 47 rifle	5.67	37.20	26.57	9.79	0.95	1.96	0.49	0.51	1.55	-

Moreover, the maximum always exceeds one.

V. DISCUSSION

The results summarized in Tables II and III suggest that a statistical detection and identification system based on the complementary use of the logarithm of signal energy and the joint entropies (one for each reference function $g_i(t)$) could be developed that would simultaneously detect discharge of firearms and classify their type. This system could be based on a hierarchical analysis beginning with $\log[E_f]$ analysis to discriminate between threat and non-threat events. If the former were indicated by the energy analysis then subsequent firearm identification would be executed using H_{f,g_i} signal receivers.

While the use of confidence ratios is a useful starting point for quantifying the statistical separation between random variables, the hierarchical analysis indicated above would require explicit knowledge of their distributions. Unfortunately, time and resource limitations precluded acquisition of a large number of firearm discharges of even a single weapon type.

Nevertheless, two conclusions seem warranted. First, and most important, is that for each firearm type there exists a receiver, either $\log[E_f]$ or one of the entropies H_{f,g_i}, that is tightly clustered with a large enough difference in mean values between different weapon types so that even a small sample of waveforms would be sufficient for statistical identification. In many cases, those where confidence ratios are larger than five, it appears that even a single weapon discharge would permit classification of a firearm into either the category of pistol or long rifle. The case where the standard deviation (σ) is less than five, identification and classification would still be possible since, unfortunately, multiple acoustic emissions from each firearm source would likely be available. In that case, statistical analysis could be based on the standard deviation of the mean ($\sigma/\sqrt{n}$), which decreases like $1/\sqrt{n}$ as the number, n of waveforms of each weapon type increases. For even a few discharges of each weapon type, $\sigma/\sqrt{n}$ would rapidly decrease so that the separations between accumulating mean values would approach five standard errors of the mean provided that the standard deviation exceeded one. To put these numbers in context suppose for the moment that the underlying distributions are normal. Then, taking the number of public elementary, middle and high schools in the United States to be 100,000,[24] and assuming that each school has 1000 rooms and that there is an acoustic event in each of these rooms once an hour for twenty four hours every day of the year, five sigma implies one false call will be made per century. While some of the numbers in this estimate may seem high, particularly the number of rooms per school, they have been chosen in order to provide an overestimate of the possible error rate.

Second, the expense of an expanded study to measure the distributions of $\log[E_f]$ and H_{f,g_i} is justified. The value of such an expanded study would lie in its use for design of an automated processing algorithm for detection of firearm discharges in public gathering places as well as the identification of firearm type.

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